

# RA3: Convex relaxations for non-convex problems in materials and industrial design

Martin Kružík

Czech Technical University, Faculty of Civil Engineering

14. 3. 2024



Co-funded by the European Union



Robotics and Advanced Industrial Production CZ.02.01.01/00/22\_008/0004590

### **Materials-Science Problem**



CuAINI shape memory alloy, IoP CAS



 $\begin{array}{l}
\text{Min} \quad T(y) = \int \mathcal{W}(\nabla y(x)) \, dx + BC \\
y \qquad \mathcal{N}
\end{array}$ 











# RA3: Convex relaxations for non-convex problems in materials and industrial design

#### RO 3.1: No relaxation gap in moment-sums-of-squares (SOS) hierarchy

**Milestone 3.1.1**: proof of non-relaxation gap for polynomial calculus of variations (COV) with convexity (micromagnetics)

**Milestone 3.1.2**: proof of non-relaxation gap for polynomial partial differential equations (PDE) from materials (multiwell problems in materials science, multiwell problems, shape memory alloys)

#### RO 3.2: Better scalability in moment-SOS hierarchy

Milestone 3.2.1: computational evidence of better scalability on benchmark problems with symmetry and sparsity

Milestone 3.2.2: new kind of sparsity (beyond correlative and term sparsity) for materials and industrial design











G3 members:

Prof. Didier Henrion (leader)



Dr. Milan Korda



Prof. Martin Kružík



Prof. Jean-Bernard Lasserre



Dr. Jakub Mareček

2 PhD students 2 post-docs

Co-funded by

the European Union



MINISTRY OF EDUCAT



## Moment SOS Hierarchy

Reformulation of non-convex problems (optimization, optimal control, COV, PDE) as infinite-dimensional linear problems on probability measures

Approximation by convex optimization (typically semidefinite optimization) problems of increasing finite dimension

Convergence guarantees based on convex duality between moments (probability theory, functional analysis) and positive polynomials expressed as sums of squares (real algebraic geometry)

Originally proposed for polynomial optimization by Lasserre (2000) and then broadly extended ...













Polynomial Optimization, Efficiency through Moments and Algebra



Interaction of Mechanics and Mathematics

Martin Kružík Tomáš Roubíček

Mathematical Methods in Continuum Mechanics of Solids

2 Springer











Thank you for your attention!





VSB TECHNICAL









Co-funded by the European Union



Robotics and Advanced Industrial Production CZ.02.01.01/00/22\_008/0004590